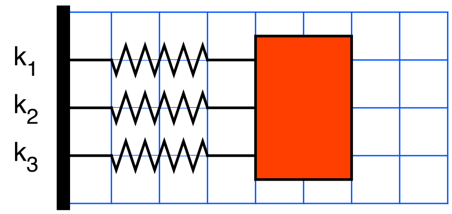
Multiple Springs

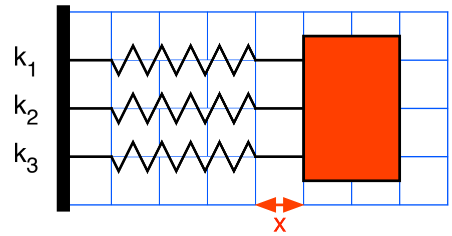
An ideal spring is one in which its force is proportional to its stretch. This is what is meant by Hooke's Law: F=kx. So what happens when there is more than one spring? One way to deal with it is to use what we will call the effective spring constant, meaning you can replace multiple springs with just one that has the correct, effective spring constant. Let's figure out how to do that.

Springs in Parallel

First, let's imagine an object that is connected to multiple springs all at once, as shown in the diagram to the right. The diagram shows 3 different springs, but we will imagine doing any number of springs. These springs are all connected to the mass in parallel (since they are all parallel to each other.) Notice that every spring is connected to the object



If we pull the object to the right one square, as shown to the right, notice how every spring stretches the same amount, which we will call x. Since each spring is exerting a force on the object, the total force on the object is just



$$F = F_1 + F_2 + F_3 + \cdots$$

By Hooke's Law, we can therefor say

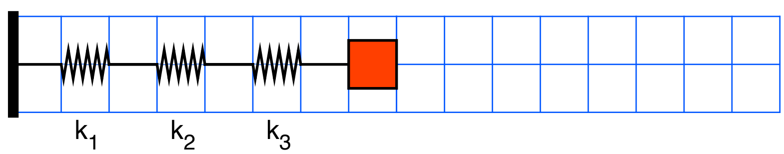
$$F = k_1x + k_2x + k_3x + \cdots = (k_1 + k_2 + k_3 + \cdots)x$$

Notice how it looks like the force on the object is the sum of the all the spring constants multiplied by the displacement – almost as if there was just one spring with a really large spring constant. This is what we mean by an effective spring constant, k_e.

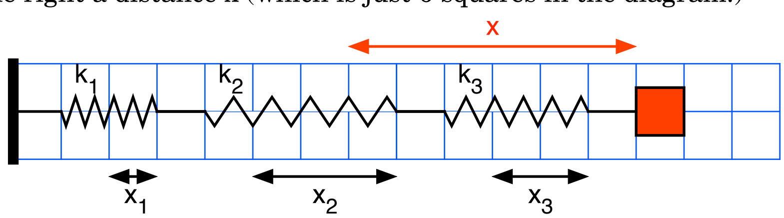
$$F = k_ex \qquad \& \qquad k_e = k_1 + k_2 + k_3 + \cdots$$

Springs in Series

We can do a similar analysis when springs are connected in series, though this is a little trickier. Series means the springs connect all in a line, as shown in the diagram below:



Notice only one of the springs is actually touching the object. That means only that one spring (k_3) directly affects the object. The diagram below shows what happens when we pull the object to the right a distance x (which is just 6 squares in the diagram.)



Because the springs have different spring constants, they all stretch different amounts, but the total amount of stretch must still add up to the displacement of the object:

$$x = x_1 + x_2 + x_3 + \cdots$$

Multiple Springs

Because of Newton's Third Law, the force in each spring must be the same. Each spring is in a tug of war with the ones adjacent to it – that is how the force on the object gets transmitted all the way to the wall (or whatever the springs are attached to) at the other end. Therefore

$$k_1 x_1 = k_2 x_2 = k_3 x_3 = \cdots$$

Because the object is only directly attached to one of the springs (k_3 in our diagram) the force acting on the object is just due to that one spring. Since the forces in the springs are all the same, we could say that the force on the object is the same as the force in any of the springs. The object itself was displaced a distance x, so the effective spring constant k_e would be defined through Hooke's Law as

$$F = k_e x \quad \& \quad k_e x = k_3 x_3 = k_2 x_2 = k_1 x_1 = \cdots$$

Notice how we can solve for each spring's displacement as

$$x_3 = \frac{k_e x}{k_3} \quad \& \quad x_2 = \frac{k_e x}{k_2} \quad \& \quad x_1 = \frac{k_e x}{k_1} \quad \& \quad \cdots$$

Since the individual spring displacements add up to the total displacement we can say

$$x = \frac{k_e x}{k_3} + \frac{k_e x}{k_2} + \frac{k_e x}{k_1} + \cdots$$

Which becomes our equation for the effective spring constant when springs are connected in series

$$\frac{1}{k_e} = \frac{1}{k_3} + \frac{1}{k_2} + \frac{1}{k_1} + \cdots$$

Summary

When multiple springs are attached to each other or to an object, we can think of the spring combination as being a single spring with an effective spring constant given by the following:

Parallel Springs: $k_e = \sum k_i$ Series Springs: $\frac{1}{k_e} = \sum \frac{1}{k_i}$

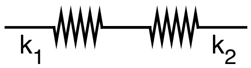
Notice how springs connected in parallel have an effective spring constant that is *larger* than the *largest* spring constant while springs connected in series have an effective spring constant that is *smaller* than the *smallest* spring constant.

Multiple Springs

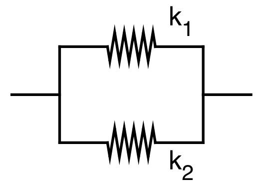
Problems:

Determine the effective spring constant for each of the following (all spring constants are given in N/m.):

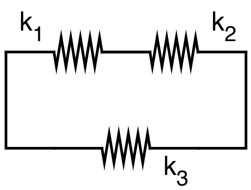
1. $k_1 = 50$ & $k_2 = 75$



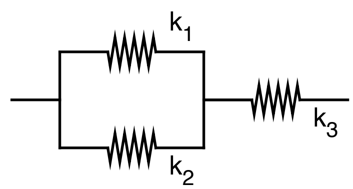
2. $k_1 = 50$ & $k_2 = 75$



3. $k_1 = 10$ & $k_2 = 30$ & $k_3 = 20$



4. $k_1 = 100$ & $k_2 = 200$ & $k_3 = 100$

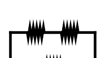


5. Imagine you have three identical springs, each with a spring constant of 10 N/m. Using all three springs, what is the largest effective spring constant you could make?

6. Imagine you have three identical springs, each with a spring constant of 10 N/m. Using all three springs, how could you make an effective spring constant of 15 N/m?

7. Imagine you have three identical springs, each with a spring constant of 10 N/m. Could you make an effective spring constant that was less than 10 N/m? (If yes, show how and give result.)

Answers: *1) 30 n/m* *2) 125 N/m* *3) 27.5 N/m* *4) 75 N/m*

5) 30 N/m, all in parallel *6)*  *7) 3.33 N/m, all in series*